## The Acoustical Structure of René Descartes's Hexachords

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As we know, the medieval system of solmization is based on hexachords which correspond to the first six degrees of the modern major key, for example, *C–D–E–F–G–A*. Its tones have been designated as solmization syllables *ut–re–mi–fa–sol–la*. Two such hexachords – *c–a* (*Hexachordum naturale*) and *g–e* (*Hexachordum durum*) – are possible in the diatonic scale. Later, one more hexachord – *f–d* (*Hexachordum molle*) – was added. If the range of a melody exceeded the hexachord, mutations – transitions from one hexachord to another – were used for its solmization.

In this study, following Isaac Newton's example, the various intervals and degrees of the hexachord are compared by means of units of the 53-division of the octave, having the size of 22.6 cents, which will be referred to as a comma unit (CU). This tuning contains practically pure fifths and thirds, the latter owing to the fact that the eighth pure fifth (815.64 cents) is only 1.95 cents larger than the natural minor sixth (813.69 cents). Therefore, all the tones of the 53-division can be designated by means of enharmonic pairs, for example *b sharp/C*, where *C* is identical to *b sharp*, the major third of *G sharp*.[1]

The French philosopher and mathematician René Descartes (1596–1650) wrote in 1618, as his first completed work, the Latin treatise *Compendium musicae*. Hexachords are dealt with in the largest chapter of Descartes's treatise, entitled "The Steps or Musical Tones". According to him, the interval *ut* to *re* must always be a minor whole-tone (9:10), *re* to *mi* a major whole-tone (8:9), *mi* to *fa* a major semitone (15:16), *fa* to *sol* a major whole-tone, and finally *sol* to *la* a minor whole-tone; see Example 1 (Näide 1). In the case of mutation from *Hexachordum molle* to *Hexachordum naturale* or from *Hexachordum naturale* to *Hexachordum durum*, two of the notes are changed and four notes remain of those that were in the previous hexachord. In the case of mutation from *Hexachordum molle* to *Hexachordum durum*, two more notes of the original four will be changed. In Example 2 (from Descartes's treatise), two new hexachords are added to the three traditional ones, namely the *B*- and *D*-hexachords. Of these, the former has no common tone with *Hexachordum durum*, nor the latter with *Hexachordum molle*.

Among the notes of the English physicist and mathematician Isaac Newton dated 1665, there is the diagram presented in Example 9, which probably represents the *B*-, *F*-, *C*-, *G*- and *D*-hexachords outlined in Example 2. In these, Descartes's figures marking string lengths are replaced by figures marking the CUs of the 53-division.

Example 10 outlines all the Descartes hexachords possible in the 53-division. In terms of the appearance of new pitches, these hexachords can be divided into three groups. The first group consists of the first seven hexachords (up to the order number –3), where each subsequent one contains two new pitches, as described by Descartes. The second group is made up of the subsequent 36 hexachords (with order numbers 4 to –21). Here each subsequent hexachord contains only one new pitch. However, one of the pitches present in the previous hexachord will be changed to its enharmonic counterpart, represented in the 53-division by the same pitch. In the third group, beginning with hexachord 22, no new pitches will appear, because all the 53 pitches of this division are already in use.

[1] The use of note names follows here the example of the 19th-century German music theorist Moritz Hauptmann, in whose treatise *Die Natur der Harmonik und Metrik* (1853) the primes and fifths of the main triads in a major key (tonic, dominant and subdominant) are marked with upper-case letters, and their thirds with lower-case letters.